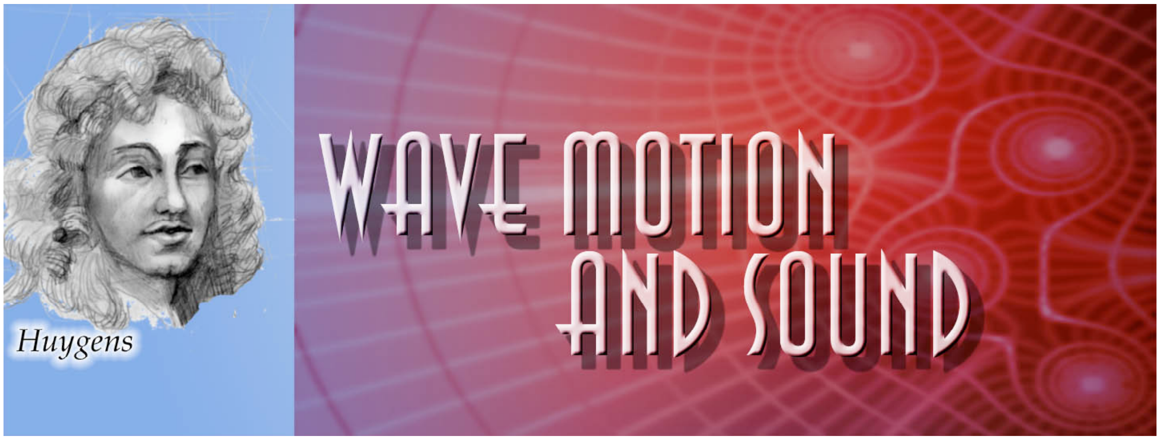


The Big Idea

Objects in motion that return to the same position after a fixed period of time are said to be in *harmonic motion*. Objects in harmonic motion have the ability to transfer some of their energy over large distances. They do so by creating waves in a medium. Imagine pushing up and down on the surface of a bathtub filled with water. Water acts as the medium that carries energy from your hand to the edges of the bathtub. Waves transfer energy over a distance without direct contact of the initial source. In this sense waves are phenomena not objects.

Key Concepts

- A *medium* is the substance through which the wave travels. For example, water acts as the medium for ocean waves, while air molecules act as the medium for sound waves.
- When a wave passes through a medium, the medium is only temporarily disturbed. When an ocean wave travels from one side of the Mediterranean Sea to the other, no actual water molecules move this great distance. Only the *disturbance* propagates (moves) through the medium.
- An object oscillating with frequency f will create waves which oscillate with the same frequency f.
- The speed v and wavelength λ of a wave depend on the nature of the medium through which the wave travels.
- There are two main types of waves we will consider: longitudinal waves and transverse waves.
- In longitudinal waves, the vibrations of the medium are in the *same direction* as the wave motion. A classic example is a wave traveling down a line of standing dominoes: each domino will fall in the same direction as the motion of the wave. A more physical example is a sound wave. For sound waves, high and low pressure zones move both forward and backward as the wave moves through them.
- In transverse waves, the vibrations of the medium are *perpendicular* to the direction of motion. A classic example is a wave created in a long rope: the wave travels from one end of the rope to the other, but the actual rope moves up and down, and not from left to right as the wave does.
- Water waves act as a mix of longitudinal and transverse waves. A typical water molecule pretty much moves in a circle when a wave passes through it.
- Most wave media act like a series of connected oscillators. For instance, a rope can be thought of as a large number of masses (molecules) connected by springs (intermolecular forces). The speed of a wave through connected harmonic oscillators depends on the

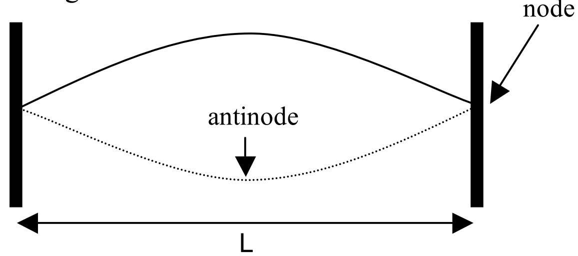
distance between them, the spring constant, and the mass. In this way, we can model wave media using the principles of simple harmonic motion.
- The speed of a wave on a string depends on the material the string is made of, as well as the tension in the string. This fact is why tightening a string on your violin or guitar will change the sound it produces.
- The speed of a sound wave in air depends subtly on pressure, density, and temperature, but is about 343 m/s at room temperature.

Key Equations

- $T = 1 / f$; period and frequency are inversely related
- $v = \lambda f$; wave speed equals wavelength times oscillation frequency
- $f_{beat} = |f_1 - f_2|$; two interfering waves create a beat wave with frequency equal to the difference in their frequencies.
- $f_n = nv / 2L$; ; a string or pipe closed at both ends or open at both ends oscillates with this frequency; n takes all integers
- $f_n = nv / 4L$; a string or pipe closed at one end oscillates with this frequency; n takes odd integers only
- $f_o = f\,(v + v_o) / (v - v_s)$; Doppler shift causes a change in observed frequency, f_o, if source (s) or observer (o) or both are moving closer
- $f_o = f\,(v - v_o) / (v + v_s)$; Doppler shift causes an observed change in frequency if source, observer or both move apart

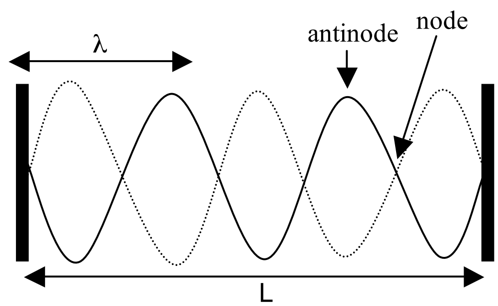
Key Applications

- *Constructive interference* occurs when two waves combine to create a larger wave. This occurs when the peaks of two waves line up.
- *Destructive interference* occurs when two waves combine and cancel each other out. This occurs when a peak in one wave lines up with a trough in the other wave.
- When waves of two different frequencies interfere, a phenomenon known as *beating* occurs. The frequency of a beat is the difference of the two frequencies.
- When a wave meets a barrier, it reflects and travesl back the way it came. The reflected wave may interfere with the original wave. If this occurs in precisely the right way, a *standing wave* can be created. The types of standing waves that can form depend strongly on the speed of the wave and the size of the region in which it is traveling.
- A typical standing wave is shown below. This is the motion of a simple jump-rope. *Nodes* are the places where the rope doesn't move at all; *antinodes* occur where the motion is greatest.



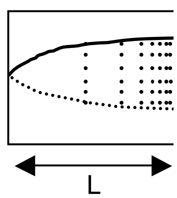
For this wave, the wavelength is $\lambda = 2L$. Since $v = \lambda f$, the frequency of oscillation is $f = v / 2L$

- *Higher harmonics* can also form. Note that each end, where the rope is attached, must always be a node. Below is an example of a rope in a 5^{th} harmonic standing wave.



In general, the frequency of oscillation is $f = nv / 2L$, where n is the number of antinodes. The thick and dotted lines represent the wave *envelope:* these are the upper and lower limits to the motion of the string.

- Importantly, each of the above standing wave examples can also apply to sound waves in a closed tube, electromagnetic waves in a wire or fiber optic cable, and so on. In other words, the standing wave examples can apply to *any* kind of wave, as long as nodes are forced at both ends by whatever is containing/reflecting the wave back on itself.
- If a node is forced at one end, but an antinode is forced at the other end, then a different spectrum of standing waves is produced. For instance, the fundamental standing sound wave produced in a tube closed at one end is shown below. In this case, the amplitude of the standing wave is referring to the magnitude of the air pressure variations.



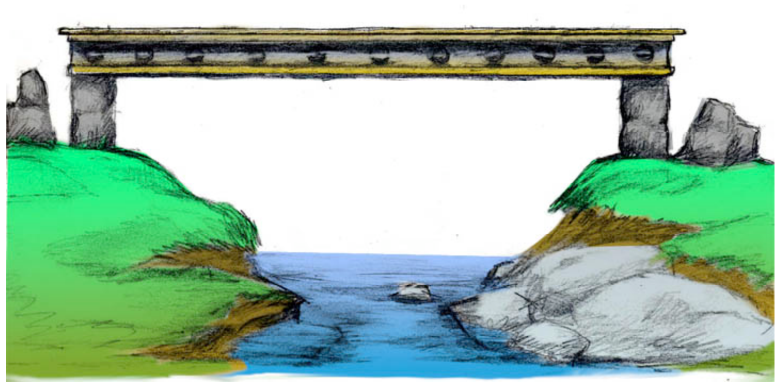
For this standing wave, the wavelength is $\lambda = 4L$. Since $v = \lambda f$, the frequency of oscillation is $f = v / 4L$. In general, the frequency of oscillation is $f = nv / 4L$, where n is always odd.

- When a source of a wave is moving towards you, the apparent frequency of the wave you detect is higher than that emitted. For instance, if a car approaches you while playing a note at 500 Hz, the sound you hear will be slightly higher. This familiar phenomenon is known as the *Doppler Effect*. The opposite occurs for a receding wave or if the observer moves or both move. There is a difference in the quantitative effect, depending on who is moving. (See the formulas under key equations above.) Note that these equations are for sound waves only. While the effect is similar for light and electromagnetic waves the formulas are not exactly the same as for sound.

Wave Motion and Sound Problem Set

1. A violin string vibrates, when struck, as a standing wave with a frequency of 260 Hz. When you place your finger on the same string so that its length is reduced to 2/3 of its original length, what is its new vibration frequency?

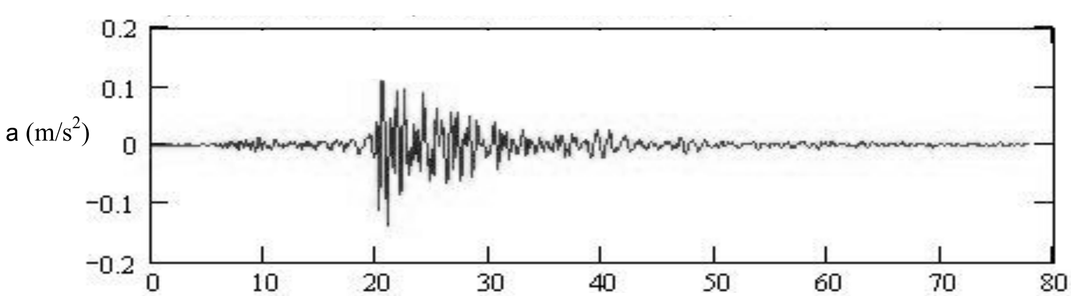
2. The simple bridge shown here oscillated up and down pretty violently four times every second as a result of an earthquake.



   a. What was the frequency of the shaking in Hz?
   b. Why was the bridge oscillating so violently?
   c. Calculate two other frequencies that would be considered "dangerous" for the bridge.
   d. What could you do to make the bridge safer?

3. The speed of water waves in deep oceans is proportional to the wavelength, which is why tsunamis, with their huge wavelengths, move at incredible speeds. The speed of water waves in shallow water is proportional to depth, which is why the waves "break" at shore. Draw a sketch to accurately portray these concepts.

4. Below you will find actual measurements of acceleration as observed by a seismometer during a relatively small earthquake.



   An earthquake can be thought of as a whole bunch of different waves all piled up on top of each other.

   a. Estimate (using a ruler) the approximate period of oscillation T of the minor aftershock which occurs around t = 40 sec.
   b. Convert your estimated period from part (a) into a frequency f in Hz.
   c. Suppose a wave with frequency f from part (b) is traveling through concrete as a result of the earthquake. What is the wavelength λ of that wave in meters? (The speed of sound in concrete is approximately v = 3200 m/s.)

5. The length of the western section of the Bay Bridge is 2.7 km.

   a. Draw a side-view of the western section of the Bay Bridge and identify all the 'nodes' in the bridge.
   b. Assume that the bridge is concrete (the speed of sound in concrete is 3200 m/s). What is the lowest frequency of vibration for the bridge? (You can assume that the towers are equally spaced, and that the central support is equidistant from both middle towers. The best way to approach this problem is by drawing in a wave that "works.")
   c. What might happen if an earthquake occurs that shakes the bridge at precisely this frequency?

6. The speed of sound v in air is approximately 331.4 m/s + 0.6T, where T is the temperature of the air in Celsius. The speed of light c is 300,000 km/sec, which means it travels from one place to another on Earth more or less instantaneously. Let's say on a cool night (air temperature 10° Celsius) you see lightning flash and then hear the thunder rumble five seconds later. How far away (in km) did the lightning strike?

7. Human beings can hear sound waves in the frequency range 20 Hz – 20 kHz. Assuming a speed of sound of 343 m/s, answer the following questions.

   a. What is the shortest wavelength the human ear can hear?
   b. What is the longest wavelength the human ear can hear?

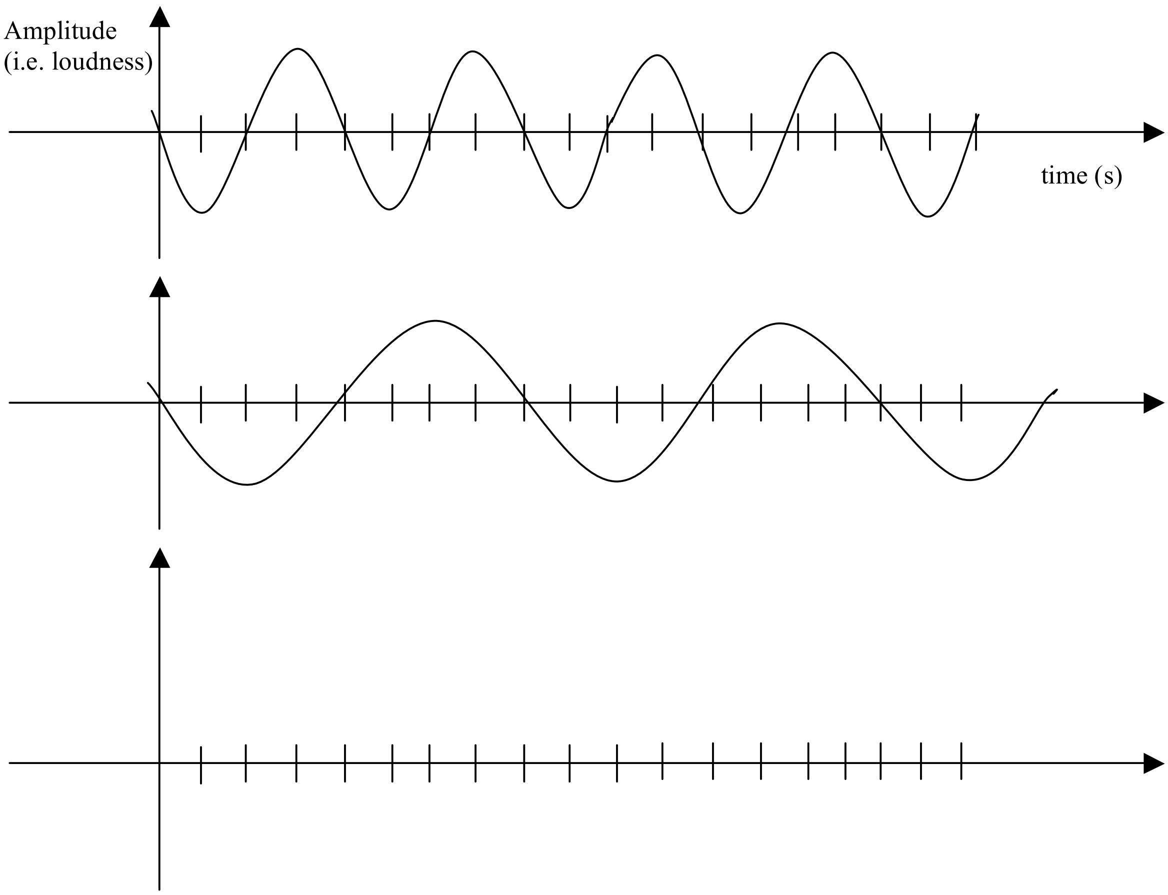
8. The speed of light c is 300,000 km/sec.

   a. What is the frequency in Hz of a wave of red light ($\lambda = 0.7 \times 10^{-6}$ m)?
   b. What is the period T of oscillation (in seconds) of an electron that is bouncing up and down in response to the passage of a packet of red light? Is the electron moving rapidly or slowly?

9. Radio signals are carried by electromagnetic waves (i.e. light waves). The radio waves from San Francisco radio station KMEL (106.1 FM) have a frequency of 106.1 MHz. When these waves reach your antenna, your radio converts the motions of the electrons in the antenna back into sound.

   a. What is the wavelength of the signal from KMEL?
   b. What is the wavelength of a signal from KPOO (89.5 FM)?
   c. If your antenna were broken off so that it was only 2 cm long, how would this affect your reception?

10. Add together the two sound waves shown below and sketch the resultant wave. Be as exact as possible – using a ruler to line up the waves will help. The two waves have different frequencies, but the same amplitude. What is the frequency of the resultant wave? How will the resultant wave sound different?



11. Aborigines, the native people of Australia, play an instrument called the Didgeridoo like the one shown above. The Didgeridoo produces a low pitch sound and is possibly the world's oldest instrument. The one shown above is about 1.3 m long and open at both ends.

    a. Knowing that when a tube is open at both ends there must be an antinode at both ends, draw the first 3 harmonics for this instrument.
    b. Derive a generic formula for the frequency of the *n*th standing wave mode for the Didgeridoo, as was done for the string tied at both ends and for the tube open at one end.

12. Reread the difference between *transverse* and *longitudinal* waves. For each of the following types of waves, tell what type it is and why. (Include a sketch for each.)

   - sound waves
   - water waves in the wake of a boat
   - a vibrating string on a guitar
   - a swinging jump rope
   - the vibrating surface of a drum
   - the "wave" done by spectators at a sports event
   - slowly moving traffic jams

13. At the Sunday drum circle in Golden Gate Park, an Indian princess is striking her drum at a frequency of 2 Hz. You would like to hit your drum at another frequency, so that the sound of your drum and the sound of her drum "beat" together at a frequency of 0.1 Hz. What frequencies could you choose?

14. A guitar string is 0.70 m long and is tuned to play an E note (f = 330 Hz). How far from the end of this string must your finger be placed to play an A note (f = 440 Hz)?

15. Piano strings are struck by a hammer and vibrate at frequencies that depend on the length of the string. A certain piano string is 1.10 m long and has a wave speed of 80 m/s. Draw sketches of each of the four lowest frequency nodes. Then, calculate their wavelengths and frequencies of vibration.

16. Suppose you are blowing into a soda bottle that is 20 cm in length and closed at one end.

   a. Draw the wave pattern in the tube for the lowest four notes you can produce.
   b. What are the frequencies of these notes?

17. You are inspecting two long metal pipes. Each is the same length; however, the first pipe is open at one end, while the other pipe is closed at both ends.

   a. Compare the wavelengths and frequencies for the fundamental tones of the standing sound waves in each of the two pipes.
   b. The temperature in the room rises. What happens to the frequency and wavelength for the open-on-one-end pipe?

18. A train, moving at some speed lower than the speed of sound, is equipped with a gun. The gun shoots a bullet forward at precisely the speed of sound, relative to the train. An observer watches some distance down the tracks, with the bullet headed towards him. Will the observer hear the sound of the bullet being fired before being struck by the bullet? Explain.

19. A 120 cm long string vibrates as a standing wave with four antinodes. The wave speed on the string is 48 m/s. Find the wavelength and frequency of the standing wave.

20. A tuning fork that produces a frequency of 375 Hz is held over pipe open on both ends. The bottom end of the pipe is adjustable so that the length of the tube can be set to whatever you please.

    a. What is the shortest length the tube can be and still produce a standing wave at that frequency?
    b. The second shortest length?
    c. The one after that?

21. The speed of sound in hydrogen gas at room temperature is 1270 m/s. Your flute plays notes of 600, 750, and 800 Hz when played in a room filled with normal air. What notes would the flute play in a room filled with hydrogen gas?

22. A friend plays an A note (440 Hz) on her flute while hurtling toward you in her imaginary space craft at a speed of 40 m/s. What frequency do you hear just before she rams into you?

23. How fast would a student playing an A note (440 Hz) have to move towards you in order for you to hear a G note (784 Hz)?

24. Students are doing an experiment to determine the speed of sound in air. The hold a tuning fork above a large empty graduated cylinder and try to create resonance. The air column in the graduated cylinder can be adjusted by putting water in it. At a certain point for each tuning fork a clear resonance point is heard. The students adjust the water finely to get the peak resonance then carefully measure the air column from water to top of air column. (The assumption is that the tuning fork itself creates an anti-node and the water creates a node.) The following data table was developed:

Frequency of tuning fork (Hz)	Length of air column (cm)	Wavelength (m)	Speed of sound (m/s)
184	46		
328	26		
384	22		
512	16		
1024	24		

    a. Fill out the last two columns in the data table.
    b. Explain major inconsistencies in the data or results.
    c. The graduated cylinder is 50 cm high. Were there other resonance points that could have been heard? If so what would be the length of the wavelength?
    d. What are the inherent errors in this experiment?